this time: hypothesis & significance

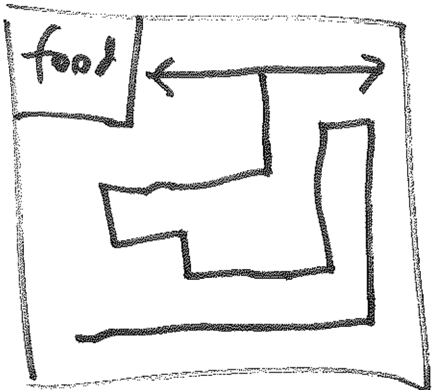
next time: testing; pitfalls

read: LN pp. L-161 + L-165

DD office hour Fri 1.15–2.15 cancelled (I'll be teaching both Fri discussion sections)

today: LN pp. L-156 → L-163

Case study: lab rats looking for food



dichotomous outcome:

turn L or R?
1 0

(1s & 0s) n = 12

(null) theory value: $p = 50\%$ ← P(turn L in pop.)

mean $\frac{10}{12} \doteq 83\% = \bar{Y} = \hat{p}$

Q_1 practsig diff. between 50% & 83%?

A_1: may (yes)

Q_2 statsig diff?

EV of $\hat{p}$ = $E_{IID}(\hat{p}) = E_{IID}(\bar{Y}) = \mu = p$

$\hat{p} = \bar{Y}$

pop.
all lab rats of some species similar to those in expt.

L = 1
R = 0

sample
stat. inf.
the observed lab rats

imag.
repeated sampling data set
all possible $\hat{p}$ values

N = ? (big)

L ?

$\begin{bmatrix} 1_s \\ \& \\ 0_s \end{bmatrix}$

actual like SRS ≐ IID

L ?

$\begin{bmatrix} 1_s \\ \& \\ 0_s \end{bmatrix}$ n = 12

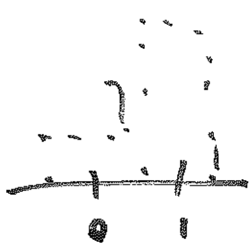
a $\hat{p}$

$\begin{bmatrix} 83\% \\ \vdots \\ \vdots \end{bmatrix}$ $M \to \infty$

mean $p = ?$

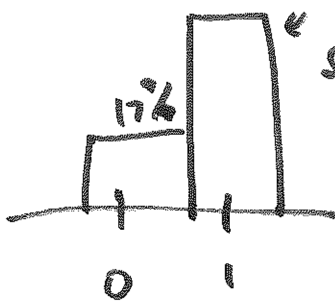
SD $\sigma = \sqrt{p(1-p)}$

hyp. IID



pop. hist.

mean $\hat{p} = \frac{10}{12} \doteq 83\%$



sample hist.

$\begin{bmatrix} \vdots \end{bmatrix}$ n = 12

mean $\hat{p} = ?$

long run mean = EV of $\hat{p}$ = p

est. long run SD = $\hat{SE}$ of $\hat{p}$ ≐ 11%

long ~~run~~ hist $\hat{SE}$ 11% ***

p

so here $\sigma = (1-0)\sqrt{p(1-p)}$ ** can't use SE formula because p unknown;

fix: est. p with $\hat{p}$: $\hat{SE}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

IID

reader formula sheet R-22

inferential summary

← ind. data ← * sample * → pop →

unknown pop. quantity of main interest	p = pop. proportion (or %) of rats that would turn L
estimate of p	$\hat{p} = \frac{10}{12} \doteq 83\%$
give-or-take for $\hat{p}$ as est. of p	$\widehat{SE}$ of $\hat{p} \doteq 11\%$
95% CI for p	$\hat{p} \pm 2\,\widehat{SE}(\hat{p}) \doteq (61\%, 100\%)$

$$SE \text{ of } \hat{p} = SE_{IID}(\hat{p}) = SE_{IID}(\bar{Y})$$

$$= \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{p(1-p)}}{\sqrt{n}} = \sqrt{\frac{p(1-p)}{n}} = (**)$$

reminder of math fact

if a pop. has only 2 values in it, (larger vs. smaller),

$$(*) \quad \text{pop. SD} = \left[\begin{pmatrix}\text{larger}\\ \text{value}\end{pmatrix} - \begin{pmatrix}\text{smaller}\\ \text{value}\end{pmatrix}\right] \sqrt{\begin{pmatrix}\text{prop.}\\ \text{of}\\ \text{larger}\end{pmatrix} \cdot \begin{pmatrix}\text{prop.}\\ \text{of}\\ \text{smaller}\end{pmatrix}}$$

4

here $\hat{SE}_{IID}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.83)(0.17)}{12}}$

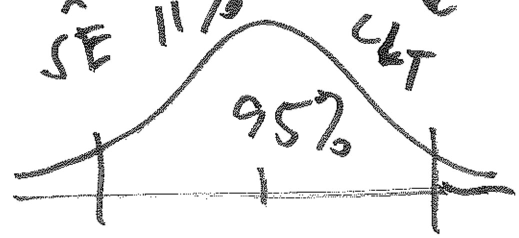
always put in decimals

$= 0.108 \doteq 11\%$

approx. long-run hist. of $\hat{p}$

(n large)

** $\hat{SE}$ 11% CLT



95%

$p - 2\hat{SE}$ $\quad p \quad$ $p + 2\hat{SE}$

-2 $\quad$ $1.96 \doteq 2$

by Mr. Neyman's logic,

$\hat{p} \pm 2\,\hat{SE}(\hat{p})$

is an approx. 95% CI for p

95% CI for p

61% $\quad$ 83% $\quad$ 105%

$\hat{p}$

truncate at 100%

my approx. 95% CI is (61%, 100%)

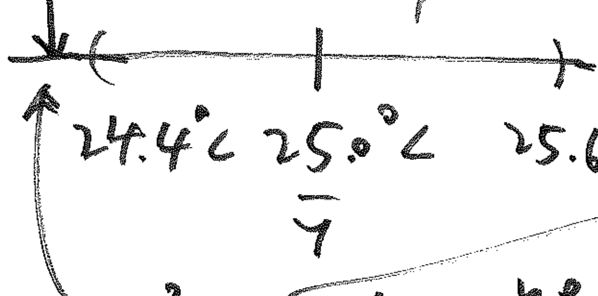
50% 61% 83% 100%

theory value $p_0 = 50\%$ is not in the CI, so data do not support null (boring) theory that $p = p_0 = 50\%$ (i.e., rats can't find food)

i.e. this diff. between 50% (theory) & 83% (data) is $\boxed{\text{statsig}}$ & is not easy to explain by unlucky sampling & is probably real

intertidal crabs revisited

μ_0 ← theory prediction



24.3°C

μ_0 not in 95% CI → theory probably wrong

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