AMS 7
26 Apr 17

This time: probability

read: DD (B) ch. 7-8
LN pp. L108 - L118

next time: ↓

today: LN pp. L-98 →

①

homework 2 not due on Fri; will announce due date later

P(1 or more T-S babies in family of 5 | both parents carriers) = P(A) = ?

HH

P(any one child normal) $= \frac{1}{4} = 25\%$

P(——— carrier) $= \frac{2}{4} = 50\%$

P(——— T-S) $= \frac{1}{4} = 25\%$

equally likely model (ELM)

Pascal & Fermat (1650)

pop ELM

$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$ at random $\begin{bmatrix} y_1 \end{bmatrix}$

$P(y_1 \text{ is odd}) = ?$

$= \frac{2}{3}$

②

# T-S kills	ELM prob.
0	1/6
1	
2	
3	
4	
5	1/6
	1

totally wrong

$\cancel{P_{ELM}(1 \text{ or more T-S})} = \frac{5}{6}$

would have been correct if ELM applied to h, j of enumeration of possible outcomes, but it doesn't

\ if S then E

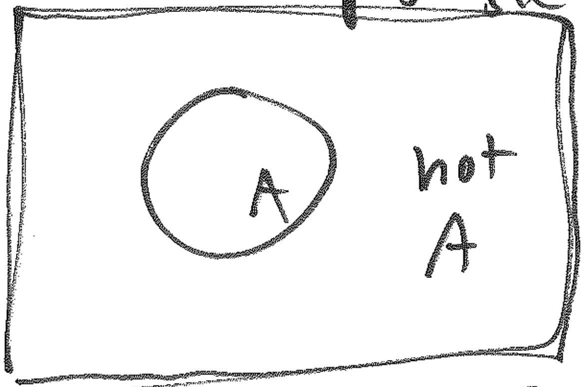
hard to know

$$P(A \text{ or } B) \overset{?}{=} P(A) \; ? \; P(B)$$

$$P(\text{not } A) \overset{?}{=} P(A)$$

$$P(A \text{ and } B) \overset{?}{=} P(A) \; ? \; P(B)$$

all possible outcomes



P(dart falls somewhere in the rectangle)

= 100% = 1

John Venn

UK,

1810-1890

$P(A) + P(\text{not } A) = 100\%$ (%) or decimal

easy rule part 2

easy rule part 1

for any event A, $0\% \le P(A) \le 100\%$

$0 \qquad\qquad 1$

$P(A) + P(\text{not } A) = 1$ so

$P(A) = 1 - P(\text{not } A)$

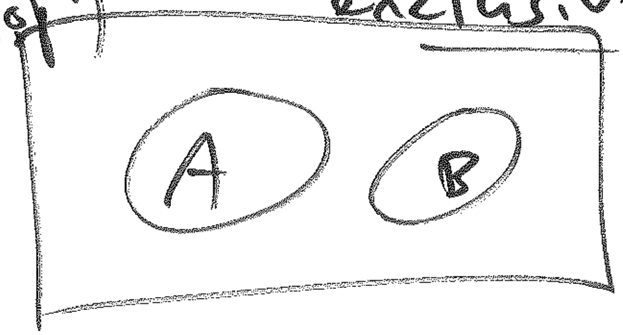
direct   indirectly

reader p. R-37

summary of basic probability rules

working with $\boxed{\text{OR}}$

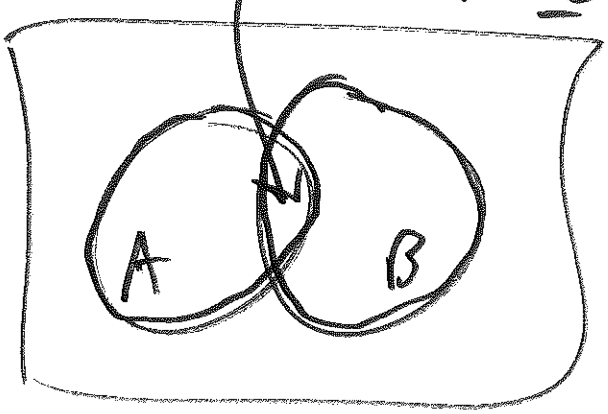
no overlap: A, B are mutually exclusive



special case of addition rule for $\boxed{\text{or}}$

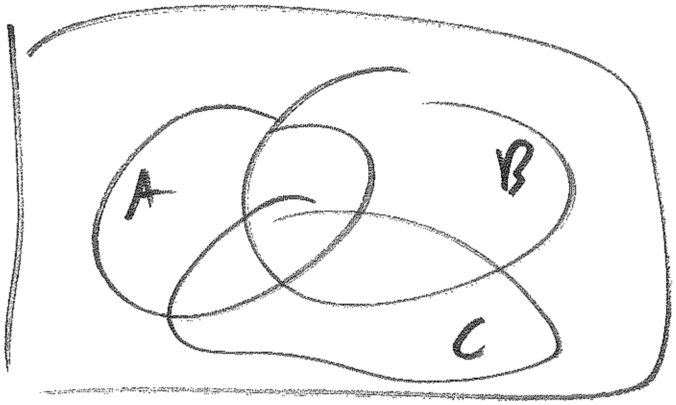
$$P(A \text{ or } B) = P(A) + P(B)$$

overlap: A and B



$$P(A \text{ or } B) = \underline{P(A)} + \underline{P(B)} - P(A \text{ and } B)$$

general addition rule for $\boxed{\text{or}}$



$P(A \text{ or } B \text{ or } C)$

$=$ complicated

working with and

$$P(A \text{ and } B) \overset{?}{=} \left\{ \begin{matrix} P(A) \\ P(B) \end{matrix} \right\}$$

$\boxed{\text{Case 1}}$

$$\begin{array}{c} \text{pop} \\ \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \end{array} \xrightarrow[\text{random}]{\text{at}} \begin{array}{c} \text{sample} \\ \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \end{array} \quad n = 2$$

at random with replacement

(Independent Identically Distributed (IID) sampling

$$\begin{array}{c} \text{pop} \\ \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \end{array} \xrightarrow{\text{IID}} \begin{array}{c} \text{sample} \\ \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \end{array} \quad n = 2$$

$$P(Y_1 = 9 \text{ and } Y_2 = 9) = ?$$

IID

$$\begin{array}{cc|ccc}
 & & & Y_2 & \\
 & & 1 & 2 & 9 \\
\hline
 & 1 & (1,1) & (1,2) & (1,9) \\
Y_1 & 2 & (2,1) & (2,2) & (2,9) \\
 & 9 & (9,1) & (9,2) & (9,9)
\end{array}$$

Q: ELM on 3×3 grid?

A: yes: each of the $3 \times 3 = 9$ possibilities is equally likely

$$P(Y_1 = 9 \text{ and } Y_2 = 9) = \frac{1}{9}$$

$$P(Y_1 = 9) = \frac{3}{9} = \frac{1}{3}$$

$$P(Y_2 = 9) = \frac{3}{9} = \frac{1}{3}$$

$\boxed{\text{our theory:}}$

$$P(Y_1 = 9 \text{ and } Y_2 = 9) = \frac{1}{9}$$

$$= P(Y_1 = 9) \cdot P(Y_2 = 9) = \frac{1}{3} \cdot \frac{1}{3}$$

$$\downarrow$$

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

turns out to be right for IID

case 2: at random without replacement (simple random sampling (SRS))

SRS

$Y_1 \backslash Y_2$	1	2	9
1	~~(1,1)~~	(1,2)	(1,9)
2	(2,1)	~~(2,2)~~	(2,9)
9	(9,1)	(9,2)	~~(9,9)~~

$P(Y_1 = 9 \text{ and } Y_2 = 9) = 0$

Q: Does ELM apply to remaining 6 possibilities?

A: yes

$P(Y_1 = 9) = \frac{2}{6} = \frac{1}{3}$

$P(Y_2 = 9) = \frac{2}{6} = \frac{1}{3}$

$P(Y_1 = 9 \text{ and } Y_2 = 9) \stackrel{?}{=} P(Y_1 = 9) \cdot P(Y_2 = 9)$

$\left(\frac{1}{3}\right) \cdot \left(\frac{1}{3}\right)$

$= \frac{1}{9} \neq 0$