this time: probability

next time: probability models for sums & means

read: DD (B) ch. 9, LN pp. L127 - 135

AMS 7 28 Apr 17

homework 2 due at 11.59pm at canvas.ucsc.edu next Wed 3 May 2017

take-home midterm handed out next Wed, due 1 week later

today: LN p. L111 →

2nd draw depends on 1st draw with SRS,

so we need a new version of prob.

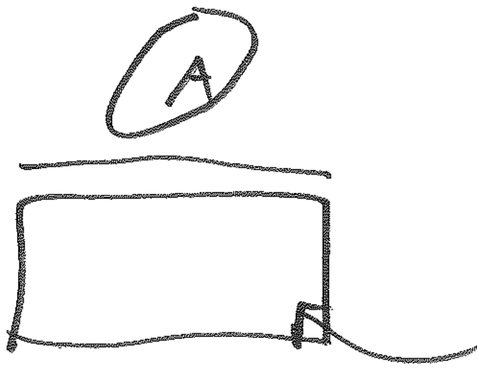
of the form P(A given B);

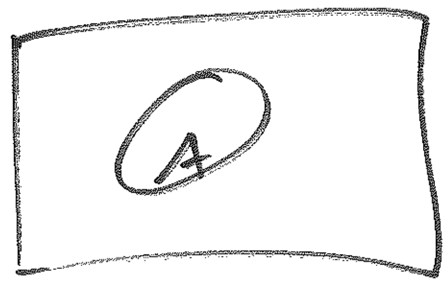
for ex., $P(Y_2 = 9 \text{ given } Y_1 = 9) =$

$$P(A \text{ given } B) = P(A \mid B)$$

conditional probability of A given B

first people to think about this were A. de Moivre (1720) & T. Bayes (1760) definition: $P(A \mid B) = ?$

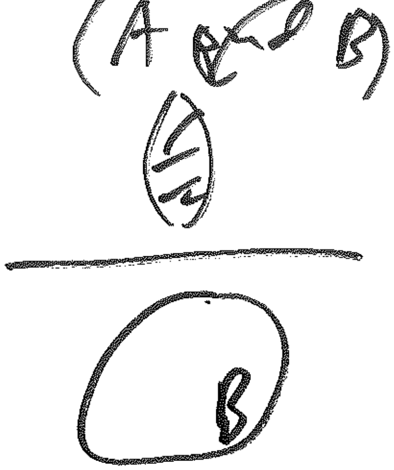
$P(A) =$ 

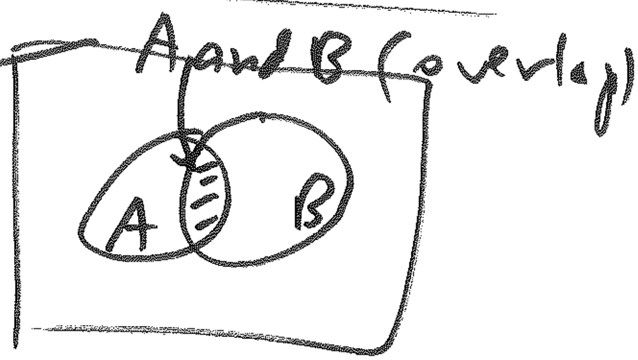


$P(\square) = 1$

$= 100\%$

$P(A \text{ given } B) =$

$P(A \mid B) =$ 



$\boxed{\text{def:}}$ As long as $P(B) > 0$,

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} \qquad (*)$$

[multiply both sides of ⊕ by $P(B)$: & reverse left & right.] ③

$$\boxed{P(A \text{ and } B) = P(B) \cdot P(A \mid B)}$$

general form of product rule for ~~[illegible]~~ working with and

ex. $P_{SRS}(Y_1 = 9 \text{ and } Y_2 = 9) =$

$\underbrace{P_{SRS}(Y_1 = 9)} \cdot P_{SRS}(Y_2 = 9 \mid Y_1 = 9) =$

$\frac{1}{3} \cdot 0 = 0$ ✓

~~true/false statements~~

def. A, B are independent if information about the truth of A does not change the chances for B,

& vice versa

using at random with replacement sampling,

the 1st & 2nd draws are independent

(this is why this kind of sampling is called IID (independent identically distributed) sampling

IID all the draws have same prob. behavior

pretty clearly, if

A, B independent, then

$$P(A \mid B) = P(A) \quad \&$$

$$P(B \mid A) = P(B)$$

this observation reconciles

IID & SRS:

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$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

$$= P(B \text{ and } A) = P(B) \cdot P(A \mid B)$$

chain rule for probability

but under IID, A, B indep.

so under IID, $P(B \mid A) = P(B)$

and $P(A \mid B) = P(A)$ +

$$P(A \text{ and } B) \overset{\text{if indep.}}{=} P(A) \cdot P(B \mid A)$$

$$= \boxed{P(A) \cdot P(B)}$$

if indep.

$$= P(B) \cdot P(A \mid B)$$

$$= \boxed{P(B) \cdot P(A)}$$

Case study: UCLA 1992

~~gender~~ ~~MLPV~~

gender	marijuana	
F ⋮ F	Y ⋮ Y	29
F ⋮ F	N ⋮ N	20
M ⋮ M	Y ⋮ Y	52
M ⋮ M	N ⋮ N	5
		106

F or M

marijuana Y or N

both categorical (qual.)

		MLP Y	MLP N	
gender	F	29	20	49
	M	52	5	57
		81	25	106

2x2 contingency table

$\boxed{Q_1}$ Is there a relationship in this data set between gender & MLP? yes

$\boxed{Q_2}$ Are gender & MLP associated? yes

$\boxed{Q_3}$ Are gender, MLP independent? no

3 ways to ask same question

Choose 1 person at random from this table ELM

$P(Y) = \frac{81}{106} \doteq 76\%$

$P(Y \mid F) = \frac{29}{49} \doteq 59\%$

$P(Y \mid M) = \frac{52}{57} \doteq 91\%$

this relationship is strong because 59% & 91% are quite different from 76%

R-51 death penalty

outcome: death penalty (DP) or not? dich

"treatment": race of defendant: defendant white (DW) or black (DB) dich

this is an obs. study: enemy in obs. studies is bias arising from PCFs

PCF: race of victim → white VW, → black VB

how control for PCF: | hold it constant

$$\left.\begin{aligned} P(DP) &= \frac{36}{326} \doteq 11\% \\ P(DP \mid DW) &= \frac{19}{160} \doteq 12\% \\ P(DP \mid DB) &= \frac{17}{166} \doteq 10\% \end{aligned}\right\} \text{top table}$$

~~white defendants getting DP more often than black defendants :~~

surprise

$$P(DP \mid VW) = \frac{30}{214} \doteq 14.0\%$$

$$P(DP \mid VW \text{ and } DW) = \frac{19}{151} \doteq 12.6\%$$

$$P(DP \mid VW \text{ and } DB) = \frac{11}{63} \doteq 17.5\%$$

$$P(DP \mid VB) = \frac{6}{112} \doteq 5.4\%$$

$$P(DP \mid VB \text{ and } DW) = \frac{0}{9} \doteq 0\%$$

$$P(DP \mid VB \text{ and } DB) = \frac{6}{103} = 5.8\%$$