this time: probability models for sums

read: DD(B) ch. 11, LN pp. L-131 → 136

AMS 7 3 May 17

next time: ——— for means

I forgot to finish the Tay-Sachs (T-S) case study: ①

P(1 or more T-S babies in family of 5, both parents carriers) = ?

Looks hard to do it directly: (1 or more) ⟷ (1 or 2 or 3 or 4 or 5)

so let's compute it indirectly:

$$P(1 \text{ or more}) = 1 - P(0 \text{ T-S babies})$$

(indep)

$$= 1 - P\left(\begin{matrix}\text{not T-S}\\\text{on 1st}\end{matrix} \text{ (and) } \begin{matrix}\text{not T-S}\\\text{on 2nd}\end{matrix} \text{ (and) } \cdots \text{ (and) } \begin{matrix}\text{not T-S}\\\text{on 5th}\end{matrix}\right)$$

(indep)

$$= 1 - P\left(\begin{matrix}\text{not T-S}\\\text{on 1st}\end{matrix}\right) \cdot P\left(\begin{matrix}\text{not T-S}\\\text{on 2nd}\end{matrix}\right) \cdots P\left(\begin{matrix}\text{not T-S}\\\text{on 5th}\end{matrix}\right)$$

(iden. dist.)

$$= 1 - \left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{4}\right)$$

(L-115)

$$= 1 - \left(1 - \frac{1}{4}\right)^5 = 1 - (0.75)^5 \doteq 0.76 \doteq 76\%$$

prob. models for sums & means

R-(5?) assume wheel fair equal prob. for all 38 outcomes & indep. of all spins

P(win on a single play of a single #) = $\frac{1}{38}$ ≐ 2.5%

P(win on a single play of a split) = $\frac{2}{38}$ = $\frac{1}{19}$ ≐ 5%

(1 draw from pop) = (1 spin of roulette wheel)

single #: (6)

		cumulative net gain
L	lost \$1	-\$1
L	lost \$1	-\$2
W	gain \$35	+\$33

\$.0526

$$\mu = \frac{(-\$1) + (-\$1) + \dots + (-\$1) + (\$35)}{38} = \frac{-\$2}{38} \doteq -\$0.05$$

(37)

population (A) — **possible spins**

sample — **the observed spins**

your net gain ($)	
−1	1
−1	2
⋮	⋮
−1	5
+35	6
−1	7
⋮	⋮
	36
	0
−1	00

$N = 38$

at random →

your net gain on a single spin ($)
−1
−1
−1
+35
−1
⋮

$n = 1{,}000$

"new" → pop. mean $\mu = \$-0.05$

pop. SD $\sigma = \$5.76$

sigma (not to be confused with capital sigma Σ sign ← summation)

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math fact: if a pop. has only 2 kinds of #s in it, (larger #) vs. (smaller #),

then $\sigma = \left[(\text{larger } \#) - (\text{smaller } \#) \right] \sqrt{(\text{proportion of larger } \#) \cdot (\text{proportion of smaller } \#)}$

(pop. mean $\mu = -\$0.05$) ↔ each time I bet $1 on a single #, I expect on average to (lose a nickel), $\mu = -\$0.05$,

give or take about $\$5.76 = \sigma$

← 37 →

$$\sigma = \sqrt{\frac{\left[(-\$1) - (-\$0.05)\right]^2 + \ldots + \left[(-\$1) - (-\$0.05)\right]^2 + \left[(+\$35) - (-\$0.05)\right]^2}{38}}$$

↑ pop. SD. not sample SDs

38 ← (N) not N − 1

here (smaller #) = −$1 (larger #) = +$35

proportion of larger # $= \frac{1}{38}$

prob. of smaller # $= \frac{37}{38}$

$\sigma = [(+35) - (-1)] \sqrt{\frac{1}{38} \cdot \frac{37}{38}} = \$5.76$