This time: probability

next time: models for sums & means

read: DD (Blch. 17) LN pp L-127 – 144

AMS 7 5 May 17

①

today: LN L-119 →

how to solve problems: find a problem whose solution you already know that's similar to the new problem you're working on, & adapt the solution to the old problem to solve the new one!

the world

your net gain after $n = 1,000$ spins of roulette wheel & $n = 1,000$ \$1 bets on a single #

is like

the sum (\$) of $n = 1,000$ IID draws from the population ⊛

the model

population — possible spins

(A): single #[6]

sample — the observed spins

(2)

imaginary (repeated sampling) data set

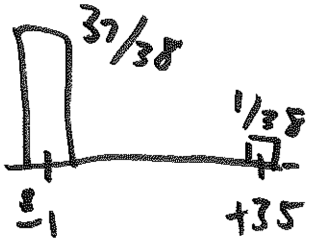
your net gain (*)

	$	
	−1	1
	−1	2
	⋮	⋮
	−1	5
N = 38	$ +35	6
	−1	7
	⋮	8
	−1	36

at random with repl. = IID

mean $\mu = \$-0.05$

SD $\sigma = \$5.76$



your net gain on this spin

$ −1, −1, ⋮, +35, ⋮

n = 1,000

sum $S = ?$

ex. $ −64

IID

IID

n = 1,000

sum $S = ?$

ex. $ 28

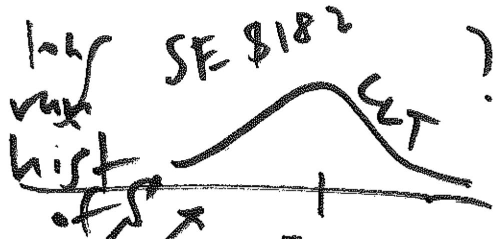
S

$ −64, −28, ⋮

M = ∞

long run mean of S = expected value of S = −$52.60

long run SD of S = standard error of S = $182



if I make n = 1,000 $1 bets on a single #, I expect at end to be behind by about $53 ($E(S) = \$-53$), give or take about $182 expect around $\frac{1{,}000}{38} \doteq 26$ wins, each winning us \$35, so we expect to win $(26)(\$35) = \$910$, but we therefore also expect to lose $1000 - 26 = 974$ times, for a total loss of $\$(-)974$; the difference is $\$910 - 974 = -\$64$

$P(\text{coming out ahead}) = P(S > \$0)$

$$\begin{pmatrix}\text{long-run}\\ \text{mean of } S\end{pmatrix} = \begin{pmatrix}\text{expected}\\ \text{value}\\ \text{of } S\end{pmatrix} = \text{EV of } S$$

math fact

$$= \boxed{E_{IID}(S) = n \cdot \mu} = (1000)(-\$0.05) \doteq -\$52.60$$

long-run SD of S = (standard error of S)

$$= SE \text{ of } S = \boxed{SE_{IID}(S) = \overset{\text{math fact}}{\frac{\sigma\sqrt{n}}{1} = \underline{\underline{\sigma\sqrt{n}}}}}$$

N	X
M	X
σ	✓ as $\sigma \uparrow$ $SE(S) \uparrow$
$M = \infty$	X
n	✓ as $n \uparrow$ $SE(S) \uparrow$

standard error of S = amount of uncertainty we have about S before the random draws have been made

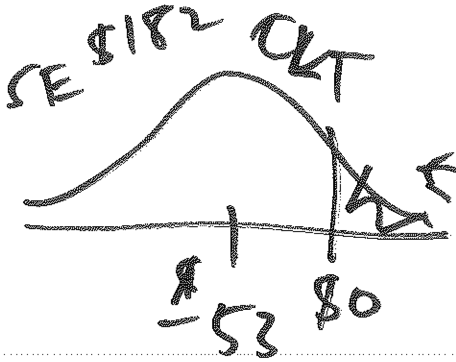
σ = uncertainty in a single draw

give-or-take = uncertainty = noise

here $\sigma\sqrt{n} = (\$5.76)\sqrt{1000}$

$\doteq \$182$

$\boxed{\text{single \#}}$
(A)



EV

long-run list of $ using $n = 1{,}000$

$\doteq 39\%$

$$\frac{\$0 - (-\$53)}{\$182} = \frac{53}{182} \doteq +0.29$$

$P\left(\text{coming out ahead with strategy } A\right) = P(S > \$0)$

$\doteq 39\%$

$\boxed{B}$ $n = 1{,}000$
S 1:7

$\downarrow$

L - 125

split

$\downarrow$

$n = 35$ single #

$E(S) = 35(-\$0.05)$
$\doteq -\$1.84$

$$\mu = \frac{(+\$17) + (+\$17) + 36(-\$1)}{38} = \frac{-\$2}{38} \doteq -\$0.05$$