April 26, 2017

This time: probability

Next time: ↓

read: DD ch. 7-8

LN pp. 108-118

today: LN pp. L-98

HW #2 will not be due on Friday, but at a later date that has not been decided yet

P(1 or more T-S ~~in~~ babies in family of 5, both parents carriers) = ? P(A) = ?

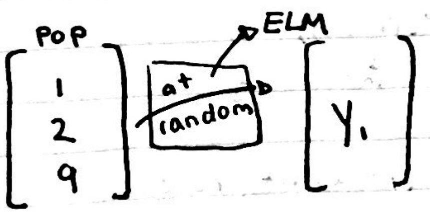
P(any one child normal $_{HH}$) = 1/4 or 25%

P(" " " carrier) = 1/2 or 50%

P(" " " T-S) = 1/4 or 25%

equally likely model (ELM)
(Pascal + Fermat (1650))

$$P(A) = \frac{\text{\# of outcomes favorable to } A}{\text{total \# of possible outcomes}}$$



$P(y_1 \text{ is odd}) = ?$

$= \frac{2}{3}$

# T-S kids	ELM prob
0	1/6
1	.
2	.
3	.
4	.
5	1/6
	1

~~P(1 or more)~~

~~P(1 or more T-S) ELM~~ = $\frac{5}{6}$

totally wrong

- would have been correct if ELM applied to possible outcomes, but it doesn't (if S then E)

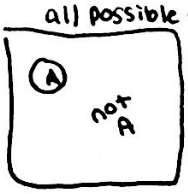
need to know

P(A or B) =? P(A) ? P(B)

P(not A) =? P(A)

P(A + B) =? P(A) ? P(B)

all possible ~~a~~ outcomes



P(dart falls somewhere in the rectangle)

= 100% → 1

percent form or decimal

John Venn UK, 1810-1890

P(A) + P(not A) = 100% } easy rule pt. 2

for ~~aaa~~ any event A

0% ≤ P(A) ≤ 100% } easy rule pt. 1

0 ≤ P(A) ≤ 1

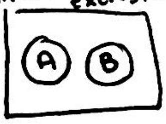
P(A) + P(not A) = 1 so P(A) = 1 − P(not A)

direct — indirectly

Reader pg. R-37 Summary of basic probability rules

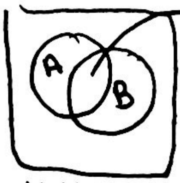
working w/ $\boxed{\text{OR}}$

mutually exclusive



P(A or B) = P(A) P(B)

Special case for the addition rule ~~of~~ for $\boxed{\text{OR}}$

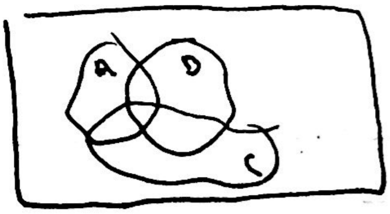


overlap: A <u>and</u> B

P(A or B)

P(A) + P(B) − P(A and B)

general addition rule for OR



P(A or B or C)

= complicated

working w/ $\boxed{\text{and}}$ P(A and B) = ? $\left\{ \begin{matrix} P(A) \\ P(B) \end{matrix} \right\}$

$$\underset{\text{pop.}}{\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}} \xrightarrow[\text{random}]{\text{at}} \underset{\text{sample}}{\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}} \quad n = 2$$

Independent Identically Distributed (IID)

- at random w/ replacement sampling

$$\underset{\text{pop.}}{\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}} \xrightarrow{\text{IID}} \underset{\text{sample}}{\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}} \quad n = 2$$

$P(Y_1 = 9$ and $Y_2 = 9) = ?$

Y_1 \ Y_2	1	2	9
1	(1,1)	(1,2)	(1,9)
2	(2,1)	(2,2)	(2,9)
9	(9,1)	(9,2)	(9,9)

Q: ELM on 3 × 3 grid?

A: yes: each of the 3 × 3 = 9 possibility is ~~equilly~~ equally likely

$P(y_1 = 9) = \frac{3}{9} = 1/3$

$P(y_2 = 9) = 3/9 = 1/3$

$P(y_1 = 9$ and $y_2 = 9)$

$= 1/9$

Theory: $P(y_1 = 9 + y_2 = 9) = 1/9$

$= P(y_1 = 9) \cdot P(y_2 = 9) = \cancel{1/9}\ \frac{1}{3} \cdot \frac{1}{3}$

~~P(A)~~ $P(A \text{ and } B) = P(A) \cdot P(B)$

turns out to be right for IID

case 2: at random w/out replacement

SRS

$p(y_1 = 9 + y_2 = 9)$
$= 0$

Y_1 \ Y_2	1	2	9
1	$\cancel{(1,1)}$	(1,2)	(1,9)
2	(2,1)	$\cancel{(2,2)}$	(2,9)
9	(9,1)	(9,2)	$\cancel{(9,9)}$

Q: Does ELM apply to remaining 6 possibilities?

A: yes

$P(y_1 = 9) = \frac{2}{6} = 1/3$

$P(y_2 = 9) = \frac{2}{6} = 1/3$

$P(y_1 = 9 \text{ and } y_2 = 9) = ?$

$P(y_1 = 9) \cdot P(y_2 = 9)$

$\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9} \neq 0$