

April 28, 2017

This time: probability

Next time: probability models for sums + means

read: DD ch. 9

L 127-135

HW #2 due Wednesday May 3rd @ 11:59pm on canvas

\* Take home midterm will be handed out next wed, and will be due one ~~week~~ week later

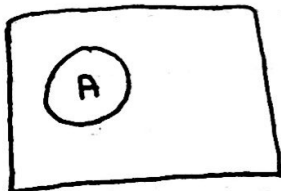
today: LN p. L (III)

2nd draw depends on first draw w/ simple random sampling  
so we need a new version of probability of the form  
 $P(A \text{ given } B)$ , for ex.,  $P(y_2 = 9 \text{ given } y_1 = 9)$

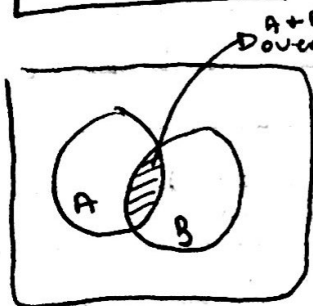
$P(A \text{ given } B) = P(A|B)$  ← conditional prob. of A given B

first people to think about this A. deMeivre (1720) + T. Bayes (1760)

def:  $P(A \text{ given } B) = ?$



$$P(A) = \frac{\text{circle A}}{\text{rectangle}} \rightarrow P(\square) = 1 = 100\%$$



$$P(A \text{ given } B) = \frac{\text{shaded area}}{\text{circle B}}$$

def: As long as  $P(A) P(B) > 0$ ,

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \quad *$$

multiply both sides of \* by  $P(B)$ ; + reverse left + right:

$$P(A \text{ and } B) = P(B) \cdot P(A/B)$$

general form of product rule for working w/ and

ex.  $P(y_1 = 9 \text{ and } y_2 = 9) =$   
SRS

$$\underbrace{P(y_1 = 9)}_{1/3} \cdot \underbrace{P(y_2 = 9 / y_1 = 9)}_{0} = 0 \checkmark$$

True/False  
state-  
ments

def: A, B are independent if info about the truth of A doesn't change the chances for B, + vice versa

using  
at random w/ replacement sampling

The first and 2nd draws are independent (this is why this kind of sampling is called IID (independent identically distributed) sampling (all the IID draws have some prob. behavior)

~~pretty~~ if A, B independent, then

$$P(A|B) = P(A) \quad \&$$
$$P(B|A) = P(B)$$

This observation reconciles IID + SRS

$$\text{IID } P(A \text{ and } B) = P(A) \cdot P(B|A) \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{chain rule for prob.}$$
$$= P(B \text{ and } A) = P(B) \cdot P(A|B)$$

but under IID A, B indep. so under IID  $P(B|A) = P(B)$

$$\text{and } P(A|B) \stackrel{\text{if indep.}}{=} P(A) \quad \& \quad P(A \text{ and } B) = P(A) \cdot P(B|A)$$

$$\boxed{= P(A) \cdot P(B)}$$
$$= P(B) \cdot P(A|B)$$
$$\boxed{= P(B) \cdot P(A)}$$

If independent

case study: UCLA 1992

Y/N	MLP		M/F
	Y	F	
	Y	F	29
	N	F	20
	Y	M	52
	N	M	5

both categorical (qual.)

	MLP		
	Y	N	
gender	29	20	49
F	52	5	57
M	81	25	106

2x2 contingency table

- Q<sub>1</sub>: is there a relationship in this data set between gender + MLP?
- Q<sub>2</sub>: Are gender and MLP associated?
- Q<sub>3</sub>: Are gender, MLP, independent?

chance 1 person to at random from table

$$P(Y) = \frac{81}{106} = 76\%$$

$$P(Y|M) = \frac{52}{57} = 91\%$$

$$P(Y|F) = \frac{29}{49} = 59\%$$

This relationship is strong b/c 59% and 91% are quite diff. from 76%.

(DP)

R-51 death penalty

outcome: death penalty or not?

(dich)

(DV)

(DB)

"treatment variable": ~~dependent~~ defendant white or black (dich)

this is an observational study: enemy is bias arising from PCFs

VB

PCF: race of ~~victim~~ victim VW

how control for PCF: hold it constant

$$\begin{aligned}
 P(OP) &= \frac{36}{326} = 11\% \\
 P(OP/DW) &= \frac{19}{160} = 12\% \\
 P(OP/DB) &= \frac{17}{166} = 10\%
 \end{aligned}
 \left. \vphantom{\begin{aligned} P(OP) \\ P(OP/DW) \\ P(OP/DB) \end{aligned}} \right\} \text{top table}$$

white defendants got OP more than black defendants

$$\begin{aligned}
 P(OP/DW) &= \frac{\cancel{19}}{\cancel{151}} \frac{36}{214} = 14\% \\
 P(OP/VW + DW) &= \frac{19}{151} = 12.6\% \\
 P(OP/VW + DB) &= \frac{11}{63} = 17.5\%
 \end{aligned}
 \left. \vphantom{\begin{aligned} P(OP/DW) \\ P(OP/VW + DW) \\ P(OP/VW + DB) \end{aligned}} \right\} \text{2nd table}$$

$$\begin{aligned}
 P(OP/VB) &= \frac{6}{112} = 5.4\% \\
 P(OP/VB + DW) &= \frac{0}{9} = 0\% \\
 P(OP/VB + DB) &= \frac{6}{103} = 5.8\%
 \end{aligned}
 \left. \vphantom{\begin{aligned} P(OP/VB) \\ P(OP/VB + DW) \\ P(OP/VB + DB) \end{aligned}} \right\} \text{3rd table}$$